



## THEORETICAL INVESTIGATIONS OF MINERAL FERTILISER DISTRIBUTION BY MEANS OF AN INCLINED CENTRIFUGAL TOOL

V. Bulgakov<sup>1</sup>, O. Adamchuk<sup>2</sup>, S. Ivanovs<sup>3</sup>

<sup>1</sup>National University of Life and Environmental Sciences of Ukraine

<sup>2</sup>National Scientific Centre of Ukraine "Institute for Agricultural Engineering"

<sup>3</sup>Latvia University of Agriculture

### Abstract

At present a great part of mineral fertilisers is introduced by means of machines equipped with centrifugal spreading tools. One of possible solutions of their improvement towards uniform fertiliser distribution around the surface of the field is the use of centrifugal spreading tools with their axes of rotation being inclined at an angle to the horizontal plane. Theoretical investigations have been conducted and a new equation obtained describing the movement of a material particle of a fertiliser along the blade of the centrifugal tool, taking into account the inclination angle of the spreading disk; and its solution is given in a closed form. The research was carried out with the use of the methods of theoretical mechanics and numerical estimations on the PC. Theoretical dependencies have been determined in order to find out absolute velocity of the particles of fertilisers at the moment they leave the spreading disk. The use of the revealed dependencies and their subsequent numerical solution on the PC provided a possibility to establish the impact of the design and kinematic parameters, the operational conditions of the inclined centrifugal distributor, particularly, the value of absolute velocity of the fertiliser particles leaving the distributor disk and their acceleration angle.

**Key words:** differential fertiliser, centrifugal spreading, absolute velocity.

### INTRODUCTION

The production efficiency of various agricultural crops depends to a considerable degree on the application of mineral fertilisers, most often introduced on the field by the surface method – spreading them using machines equipped with centrifugal spreading tools, were described by BIOCCA (2013). The use of these machines has indisputable advantages over the other similar machines. Yet they need further improvement, which concerns, first of all, the uniformity of fertiliser distribution around the surface of the field and depends on the design and kinematic parameters of their operating tools.

It is well-known that the working width of the machine for the application of mineral fertilisers by a centrifugal method depends on the value of absolute velocity  $V_{AC}$  of fertilisers leaving the surface of its spreading tool and angle  $\alpha_{AC}$  between the vector of the latter and the horizontal plane. Value  $V_{AC}$  depends on the geometric parameters and the kinematic conditions of the operation of the centrifugal spreading tool, as well as on physical and mechanical properties of mineral fertilisers.

As a result of earlier conducted investigations, optimisation of the geometric parameters of the centrifugal spreading tool was carried out, taking into account the

physical and mechanical properties of mineral fertilisers, were described by ADAMCHUK (2004). Besides, it was established that increase in the kinematic conditions of the operation of the centrifugal spreading tool is limited by the strength of the granules of the fertilisers. Therefore, when using the common construction materials and kinds of fertilisers, the possibility to increase the working width of the machines by increasing velocity  $V_{AC}$  is exhausted.

In order to optimise the centrifugal spreading tool with an inclined axis of rotation, it is necessary to have a methodology which would ensure determination of absolute velocity of fertilisers leaving its surface and the angle between the vector of the latter and the horizontal plane depending on the parameters and conditions of operation of the centrifugal spreading tool, as well as the physical and mechanical properties of mineral fertilisers. It is known that an increase in the distribution width of mineral fertilisers using the centrifugal spreading tool is possible by reaching rational values of angle  $\alpha_{AC}$ , were described by ADAMCHUK (2005). The obtained results of research witness that the rational values of angle  $\alpha_{AC}$  are situated within the limits 30°...35°. At the same time it was established that the existing centrifugal spreading tools can

reach the values of angle  $\alpha_{AC}$  not higher than  $15.7^\circ$ , were described by ADAMCHUK (2002). The centrifugal spreading tools with an inclined axis of rotation ensure higher optimal values of angle  $\alpha_{AC}$ , were described by ADAMCHUK (2005). The familiar methodologies by using of which one can determine absolute velocity of a fertiliser particle leaving the centrifugal spreading tool with a vertical axis of rotation, were described by ADAMCHUK (2010) and the methodology which allows determination of absolute velocity of a fertiliser particle leaving the centrifugal spreading tool with a horizontal axis of rotation, were described

## MATERIALS AND METHODS

The design of the centrifugal spreading tool developed by us with an inclined axis of rotation comprises a flat disk which has blades radially installed on its working surface and is cinematically joined with the drive unit of rotary movement. Besides, the axis of rotation of the centrifugal spreading tool is arranged at an angle  $\alpha$  to the horizontal plane.

For such a centrifugal distributor of mineral fertilisers we will build an estimated mathematical model of the movement of a material particle along its radially installed blade so that the axis of rotation of the spreading tool has an inclination. For this purpose, first of all, we will compose an equivalent scheme in which we will show the material particle moving along the blade of the inclined spreading disk, and we will show the forces acting upon it (Fig. 1). There:

$M$  – the initial position of the fertiliser particle on the blade, point  $S$  – the current position of the fertiliser particle on the blade, point  $O$  – the centre of rotation of the centrifugal spreading tool.

Further, to simplify the analytical solution of the present task, we make the following assumptions:

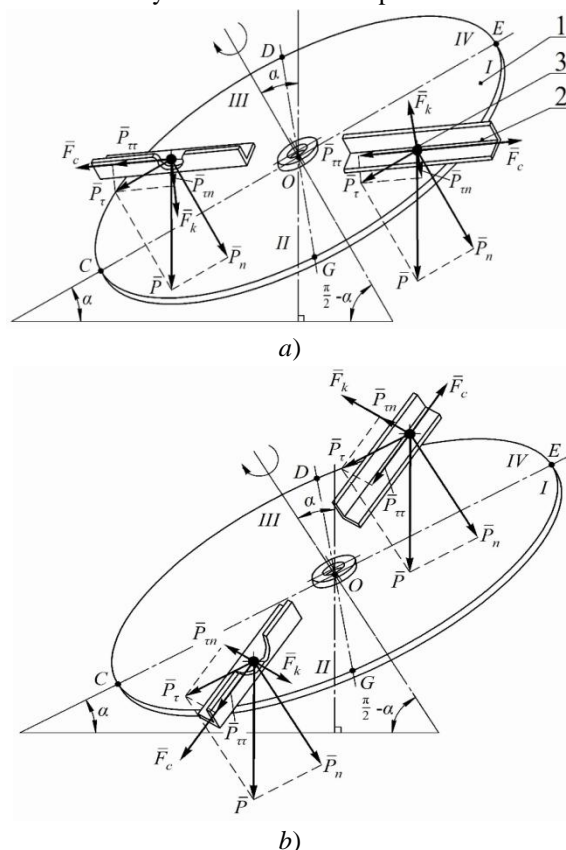
- the coefficient of friction of the fertiliser particles against the surface of the blade has a constant value;
- the character of the movement of each fertiliser particle is the same, and it corresponds to the character of the movement of the entire mass of fertilisers along the blade;
- the fertiliser particle is moving along a section of the blade which is common for the vertical wall of the blade and its bottom, without a rolling motion;
- the thickness of the blade and the diameter of the fertiliser particle are neglected.

An essential difference of dispersion of the particles of mineral fertilisers using the centrifugal spreading tool with an inclined axis of rotation, in contrast to the horizontal one, is that here there are basic differences

by VASILENKO (1960, 1996), BULGAKOV (2014) do not consider determination of absolute velocity of a fertiliser particle leaving the centrifugal spreading tool with an inclined axis of rotation to the horizontal plane.

The aim of the investigation is to obtain new analytical dependencies in order to discover the impact of the design and kinematic parameters and modes of operation of an inclined centrifugal distributor of mineral fertilisers, particularly, the value of absolute velocity of the fertiliser particles leaving the distributor disk, as well as their acceleration angle.

in the position of vectors of forces applied to the material particle depending on at which place of the inclined disk mineral fertilisers are supplied and gripped by the blades: at the upper part of the inclined disk or its lower part, at the right side of the axis of rotation or at its left side. This circumstance should also be considered in analytical solution of the present task.



**Fig. 1.** – An equivalent scheme of the movement of a fertiliser particle along the blade of the spreading disk inclined at angle  $\alpha$  to the horizon (*a, b* – respectively, a fertiliser particle is moving along the blade within the limits of sectors I, III and II, IV): 1 – the disk; 2 – the blade; 3 – a fertiliser particle

At first let us write an equation in order to determine absolute velocity  $V_{AC}$  of a fertiliser particle leaving the centrifugal spreading tool. It will be equal to:

$$V_{AC} = \sqrt{V_{rc}^2 + V_{NC}^2}, \quad (1)$$

where:  $V_{rc}$  – relative velocity of the movement of a fertiliser particle at the moment it leaves the surface of the centrifugal spreading tool  $\text{m}\cdot\text{s}^{-1}$ ;  $V_{NC}$  – transportation velocity of the movement of a fertiliser particle at the moment it leaves the surface of the centrifugal spreading tool  $\text{m}\cdot\text{s}^{-1}$ .

In this case the transportation velocity  $V_{NC}$  of the movement of a fertiliser particle at the moment it leaves the surface of the centrifugal spreading tool can be determined by means of such a dependency:

$$V_{NC} = \omega R, \quad (2)$$

where  $\omega$  – angular velocity of the centrifugal spreading tool,  $\text{s}^{-1}$ ;  $R$  – the radius of the centrifugal spreading tool, m.

In order to determine  $V_{AC}$ , it is necessary, first of all, to have a dependency for the estimation of relative velocity  $V_{rc}$ . Due to the fact that the projection of the component of force  $\bar{P}$  of weight of the fertiliser particle upon segment  $AB$  in the process of its movement along the blade changes the direction of the vector, it is purposeful to divide the centrifugal spreading tool into sectors in such a way that the direction of the vector in the process of its movement did not change within the limits of each sector. Fulfilling this, we obtain four mutually equal sectors:  $EOG$  – I;  $GOC$  – II;  $COD$  – III;  $DOE$  – IV (Fig. 2).

Segments  $EC$  and  $DG$  are mutually perpendicular diameters of the flat disk but segment  $EC$  forms angle  $\alpha$  with the horizontal plane.

Let us determine the values of forces acting upon the fertiliser particle  $M$  and write an equation for the resultant force  $F_r$  under the impact of which this particle will move along the blade (Fig. 1, 2).

Since the movement of the material particle of the mineral fertiliser proceeds in a rectilinear direction along the surface of the blade, we will write this equation in the form of projections upon the axis which coincides with the surface of the blade itself:

$$F_r = F_c \pm P_{\tau\tau} - f_f F_k - f_f P_n - f_f P_{\tau n}, \quad (3)$$

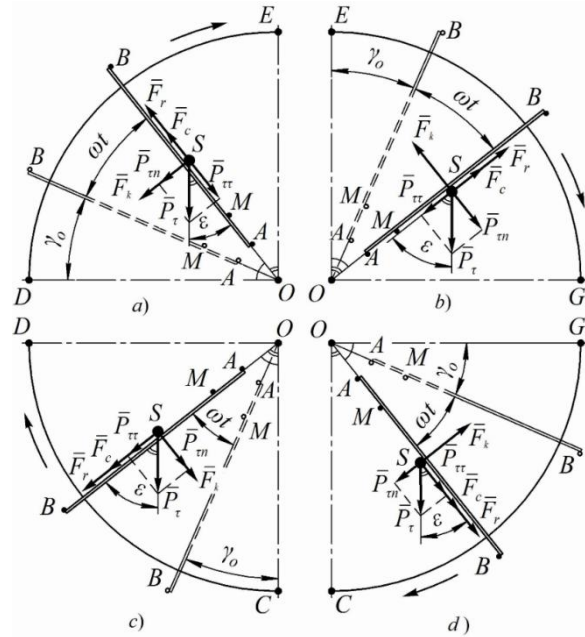
where:  $f_f$  – the friction coefficient of the fertiliser particle  $M$  against the surface of the blade.

Let us determine the values of forces applied to particle  $M$ , which constitute expression (3).

The resultant force under the impact of which the fertiliser particle  $M$  is moving along the blade is defined as:

$$F_r = m \frac{d^2 L}{dt^2}, \quad (4)$$

where:  $m$  – the mass of the fertiliser particle, kg;  $L$  – the path covered by the fertiliser particle moving along the blade, m;  $t$  – the time (duration) of the movement of the fertiliser particle along the blade, s.



**Fig. 2.** – A scheme of the distribution of the resultant force under the impact of which the fertiliser particle is moving along the blade of the centrifugal spreading tool: a, b, c, d – respectively, the fertiliser particle is moving along the blade within the limits of sectors IV, I, III and II

The centrifugal force  $\bar{F}_c$  of inertia is determined by means of the expression:

$$F_c = mr\omega^2, \quad (5)$$

where:  $r$  – the distance from the centre of rotation of the centrifugal spreading tool before the current position of the fertiliser particle on the blade, m;  $\omega$  – angular velocity of the centrifugal spreading tool,  $\text{s}^{-1}$ .

The projection of the component of force  $\bar{P}_\tau$  of the weight of the fertiliser particle upon segment  $AB$  is defined as:

$$P_{\tau\tau} = P_\tau \cos \varepsilon, \quad (6)$$

where:  $\varepsilon$  – the angle between the component of the force of weight  $\bar{P}_\tau$  and its projection upon segment  $AB$ , rad.



The component of force  $\bar{P}_r$  of the weight of the fertiliser particle acting along the surface of the disk, parallel to segment  $EC$ , will be defined by such an expression:

$$P_r = P \sin \alpha, \quad (7)$$

where:  $\alpha$  – the angle between the axis of rotation of the centrifugal spreading tool and the vertical plane, rad.

Force  $\bar{P}$  of the weight of the fertiliser particle will be equal to:

$$P = mg, \quad (8)$$

where:  $g$  – the free fall acceleration,  $\text{m}\cdot\text{s}^{-2}$ .

The Coriolis force of inertia is defined by the expression:

$$F_k = 2m\omega \frac{dL}{dt}. \quad (9)$$

The component of the force of the weight of the fertiliser particle acting along the normal to the bottom of the blade has such an appearance:

$$m \frac{d^2 L}{dt^2} = mr\omega^2 \pm mg \sin \alpha \cdot \cos \varepsilon - f_f \left( 2m\omega \frac{dL}{dt} + mg \cos \alpha + mg \sin \alpha \cdot \sin \varepsilon \right) \quad (12)$$

As it follows from Fig. 2, depending on the sector in which fertilisers are supplied onto the surface of the centrifugal spreading tool, the values of angle  $\varepsilon$  between the component of the vector of the force of weight  $\bar{P}_r$  and its projection onto segment  $AB$  will be different, and the values of this angle are defined by such four expressions:

$\varepsilon = \gamma_o + \omega t$  – for the case when fertilisers come onto the surface of the centrifugal spreading tool within the limits of sector I,

where:  $\gamma_o$  – the angle formed by segments  $OE$  and  $OB$  at the contact moment of the fertiliser particle with the blade, rad;

$\varepsilon = \frac{\pi}{2} - (\gamma_o + \omega t)$  – for the case when fertilisers come onto the surface of the centrifugal spreading tool within the limits of sector II,

where:  $\gamma_o$  – the angle formed by segments  $OG$  and  $OB$  at the contact moment of the fertiliser particle with the blade, rad;

$\varepsilon = \gamma_o + \omega t$  – for the case when fertilisers come onto the surface of the centrifugal spreading tool within the limits of sector III,

$$P_n = P \cos \alpha. \quad (10)$$

The projection of the component of force  $P_r$  of the weight of the fertiliser particle upon the normal to segment  $AB$  will be equal to:

$$P_{rn} = P_r \sin \varepsilon. \quad (11)$$

It should be remarked that in case the particles (the flow) of fertilisers come onto the surface of the centrifugal spreading tool within the limits of sector I or IV, then into expression (3) before force  $P_{rn}$  one should put the sign “–”, if within the limits of sector II or III, then it is necessary to put the sign “+” before the symbol of this force.

By substituting the values of forces defined by expressions (4)–(11) into equation (3) we will obtain a differential equation of the movement of the fertiliser particle  $M$  along the blade of the centrifugal spreading tool, inclined at angle  $\alpha$  to the horizon:

where:  $\gamma_o$  – the angle formed by segments  $OC$  and  $OB$  at the contact moment of the fertiliser particle with the blade, rad;

$\varepsilon = \frac{\pi}{2} - (\gamma_o + \omega t)$  – for the case when fertilisers come onto the surface of the centrifugal spreading tool within the limits of sector IV,

where:  $\gamma_o$  – the angle formed by segments  $OD$  and  $OB$  at the contact moment of the fertiliser particle with the blade, rad.

Further we will write an equation for distance  $r$  from the centre of rotation of the centrifugal spreading tool to the current position of the fertiliser particle  $S$  on the blade. It is defined by means of the expression:

$$r = r_o + L, \quad (13)$$

where:  $r_o$  – the radius of the supply of the fertiliser particle onto the centrifugal spreading tool, m.

By substituting into expression (12) the value of distance  $r$  and making a series of transformations we will obtain:

$$\frac{d^2 L}{dt^2} = \omega^2 r_o + \omega^2 L \pm g \sin \alpha \cdot \cos \varepsilon - 2f_f \omega \frac{dL}{dt} - f_f g \cos \alpha - f_f g \sin \alpha \cdot \sin \varepsilon. \quad (14)$$



Let us consider a case when the fertilisers are supplied onto the surface of the centrifugal spreading tool within the limits of sector II (*GOC*). Then equation (14) will have the appearance:

$$\frac{d^2L}{dt^2} + 2f_f\omega \frac{dL}{dt} - \omega^2 L = (\omega^2 r_o - f_f g \cos \alpha) + g \sin \alpha \cdot \sin(\gamma_o + \omega t) - f_f g \sin \alpha \cdot \cos(\gamma_o + \omega t). \quad (15)$$

In such a way a linear differential equation of the second order is obtained with constant coefficients and the right-side part.

Let us solve the obtained differential equation (15). Its characteristic equation will look like this:

$$\lambda^2 + 2f_f\omega\lambda - \omega^2 = 0, \quad (16)$$

but its roots will correspondingly be equal:

$$\lambda_1 = \omega(\sqrt{f_f^2 + 1} - f_f), \quad \lambda_2 = \omega(-\sqrt{f_f^2 + 1} - f_f). \quad (17)$$

Let us write a general solution  $\bar{L}$  of equation (15) without the right side:

$$\bar{L} = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}, \quad (18)$$

where:  $C_1$  and  $C_2$  – arbitrary constants.

Further we will find a specific solution  $L^*$  of equation (15).

Let us introduce the following designations:

$$\omega^2 r_o - f_f g \cos \alpha = K, \quad g \sin \alpha = U. \quad (19)$$

Then, taking into account the accepted designations, the right side of the differential equation (15) will have such an appearance:

$$K + U \sin(\gamma_o + \omega t) - f_f U \cos(\gamma_o + \omega t) \quad (20)$$

In this case we will look for the specific solution of the heterogeneous equation in the following way:

$$L^* = W \sin(\gamma_o + \omega t) + Z \cos(\gamma_o + \omega t) + J \quad (21)$$

where:  $W$ ,  $Z$  and  $J$  – the unknown coefficients.

These unknown coefficients are defined using the method of indefinite coefficients. For this, we will twice differentiate the specific solution (21). We have:

$$\frac{dL^*}{dt} = \omega W \cos(\gamma_o + \omega t) - \omega Z \sin(\gamma_o + \omega t) \quad (22)$$

$$\frac{d^2L^*}{dt^2} = -\omega^2 W \sin(\gamma_o + \omega t) - \omega^2 Z \cos(\gamma_o + \omega t) \quad (23)$$

Let us substitute the obtained expressions (22) and (23) into equation (15). We will have:

$$\begin{aligned} & -\omega^2 W \sin(\gamma_o + \omega t) - \omega^2 Z \cos(\gamma_o + \omega t) + \\ & + 2f_f\omega [\omega W \cos(\gamma_o + \omega t) - \omega Z \sin(\gamma_o + \omega t)] - \\ & - \omega^2 [W \sin(\gamma_o + \omega t) + Z \cos(\gamma_o + \omega t) + J] = \\ & = K + U \sin(\gamma_o + \omega t) - f_f U \cos(\gamma_o + \omega t). \end{aligned} \quad (24)$$

Performing the necessary transformations of expression (24), we will obtain:

$$\begin{aligned} & -\omega^2 W \sin(\gamma_o + \omega t) - \omega^2 Z \cos(\gamma_o + \omega t) + \\ & + 2f_f\omega^2 W \cos(\gamma_o + \omega t) - 2f_f\omega^2 Z \sin(\gamma_o + \omega t) - \\ & - \omega^2 W \sin(\gamma_o + \omega t) - \omega^2 Z \cos(\gamma_o + \omega t) - \omega^2 J = \\ & = K + U \sin(\gamma_o + \omega t) - f_f U \cos(\gamma_o + \omega t). \end{aligned} \quad (25)$$

Further we equate the coefficients at the corresponding trigonometric functions. We have:

$$\left. \begin{aligned} -\omega^2 W - 2f_f\omega^2 Z - \omega^2 W &= U, \\ -\omega^2 Z + 2f_f\omega^2 W - \omega^2 Z &= -f_f U, \\ -\omega^2 J &= K, \end{aligned} \right\} \quad (26)$$

$$\left. \begin{aligned} -2\omega^2 W - 2f_f\omega^2 Z &= U, \\ -2\omega^2 Z + 2f_f\omega^2 W &= -f_f U, \\ -\omega^2 J &= K. \end{aligned} \right\} \quad (27)$$

From the obtained system of linear equations (27) in relation to unknowns  $R$ ,  $S$  and  $T$  we find the values of these unknown coefficients:

$$J = -\frac{K}{\omega^2}, \quad Z = 0, \quad W = -\frac{U}{2\omega^2}. \quad (28)$$

Substituting the values of the obtained coefficients (28) into expression (21), we obtain a specific solution of the heterogeneous differential equation:

$$L^* = -\frac{U}{2\omega^2} \sin(\gamma_o + \omega t) - \frac{K}{\omega^2}. \quad (29)$$

The general solution of the differential equation (15) can be written like this:

$$L = \bar{L} + L^* = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} - \frac{U}{2\omega^2} \sin(\gamma_o + \omega t) - \frac{K}{\omega^2}. \quad (30)$$

The arbitrary constants  $C_1$  and  $C_2$  are found from the following initial conditions:

$$\text{at } t=0: \quad L=0, \quad \frac{dL}{dt}=0.$$



For this, we differentiate by  $t$  expression (30). We will have:

$$\frac{dL}{dt} = \lambda_1 C_1 e^{\lambda_1 t} + \lambda_2 C_2 e^{\lambda_2 t} - \frac{U}{2\omega} \cos(\gamma_o + \omega t) \quad (31)$$

Using the presented initial conditions, we obtain the following system of algebraic equations in relation to unknowns  $C_1$  and  $C_2$ :

$$\left\{ \begin{array}{l} C_1 + C_2 - \frac{U}{2\omega^2} \sin \gamma_o - \frac{K}{\omega^2} = 0, \\ \lambda_1 C_1 + \lambda_2 C_2 - \frac{U}{2\omega} \cos \gamma_o = 0. \end{array} \right. \quad (32)$$

## RESULTS AND DISCUSSION

In order to arrive at a final solution of the differential equation (15) and to establish the rule of the movement of a fertiliser particle along the blade of the centrifugal spreading tool, inclined at  $\alpha$  to the horizon (35), we substitute the obtained values (33) and (34) of the arbitrary constants  $C_1$  and  $C_2$  into expression (30):

$$\begin{aligned} L = & \left[ \frac{U \cos \gamma_o}{2\omega(\lambda_1 - \lambda_2)} - \frac{U \lambda_2 \sin \gamma_o}{2\omega^2(\lambda_1 - \lambda_2)} - \frac{K \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} \right] e^{\lambda_1 t} + \\ & + \left[ -\frac{U \cos \gamma_o}{2\omega(\lambda_1 - \lambda_2)} + \frac{U \lambda_2 \sin \gamma_o}{2\omega^2(\lambda_1 - \lambda_2)} + \frac{K \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} + \right. \\ & \left. + \frac{U \sin \gamma_o}{2\omega^2} + \frac{K}{\omega^2} \right] e^{\lambda_2 t} - \frac{U \sin(\gamma_o + \omega t)}{2\omega^2} - \frac{K}{\omega^2}. \end{aligned} \quad (35)$$

After substitution of expressions (33) and (34) into expression (31), we obtain the rule about the change of velocity  $V_r$  in relation to the movement of the fertiliser particle along the blade at an arbitrary moment of time  $t$ :

$$\begin{aligned} V_r = \frac{dL}{dt} = & \left[ \frac{U \cos \gamma_o}{2\omega(\lambda_1 - \lambda_2)} - \frac{U \lambda_2 \sin \gamma_o}{2\omega^2(\lambda_1 - \lambda_2)} - \frac{K \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} \right] \lambda_1 e^{\lambda_1 t} + \\ & + \left[ -\frac{U \cos \gamma_o}{2\omega(\lambda_1 - \lambda_2)} + \frac{U \lambda_2 \sin \gamma_o}{2\omega^2(\lambda_1 - \lambda_2)} + \frac{K \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} + \right. \\ & \left. + \frac{U \sin \gamma_o}{2\omega^2} + \frac{K}{\omega^2} \right] \lambda_2 e^{\lambda_2 t} - \frac{U}{2\omega} \cos(\gamma_o + \omega t). \end{aligned} \quad (36)$$

In order to determine the time  $t_1$  of the movement of a fertiliser particle along the blade from the point of its supply (point  $M$ ) to the point of its leaving the blade (point  $B$ ), it is necessary to replace  $L$  in expression (35) by its value  $L = R - r_o$ , which determines the distance between points  $M$  and  $B$ , and to solve the obtained equation in relation to time  $t_1$ . By substitut-

ing the obtained value of time  $t_1$  into equation (36), we obtain value  $V_{rc}$  of the relative velocity of the movement of the fertiliser particle at the moment when it leaves the surface of the spreading disk.

$$C_1 = \frac{U \cos \gamma_o}{2\omega(\lambda_1 - \lambda_2)} - \frac{U \lambda_2 \sin \gamma_o}{2\omega^2(\lambda_1 - \lambda_2)} - \frac{K \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} \quad (33)$$

And

$$C_2 = -\frac{U \cos \gamma_o}{2\omega(\lambda_1 - \lambda_2)} + \frac{U \lambda_2 \sin \gamma_o}{2\omega^2(\lambda_1 - \lambda_2)} + \frac{K \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} + \frac{U \sin \gamma_o}{2\omega^2} + \frac{K}{\omega^2} \quad (34)$$

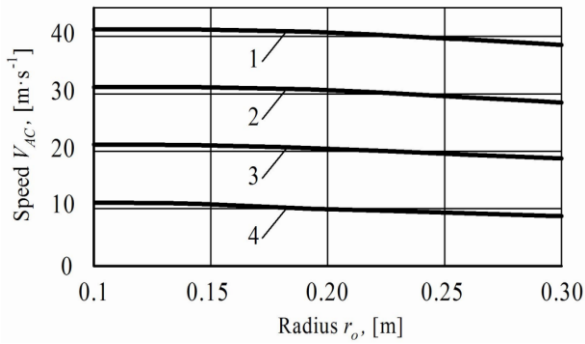
In such a way, taking into account expression (1), we have a possibility to determine the value of absolute velocity  $V_{AC}$  at the moment it leaves the surface of the spreading disk, when fertilisers are supplied onto the surface of the disk within the limits of sector II (GOC).

Using the obtained analytical expressions, in accordance with the worked out programme, numerical estimations were performed on the PC, which provided a possibility to determine the impact of  $\omega$ ,  $r_o$  and  $\alpha$  upon  $V_{AC}$ .

It has been established that increase in the value of  $\alpha$  from  $0^\circ$  to  $90^\circ$  leads to the change of  $V_{AC}$  not more than by  $0.1 \text{ m}\cdot\text{s}^{-1}$ . The impact of  $\omega$  and  $r_o$  upon  $V_{AC}$  is presented in Fig. 3.

As it is evident from the graphs in Fig. 3, at  $\omega = 104.6 \text{ s}^{-1}$  increase of  $r_o$  from 0.1 m to 0.3 m leads to a decrease of  $V_{AC}$  from  $41.32 \text{ m}\cdot\text{s}^{-1}$  to  $38.21 \text{ m}\cdot\text{s}^{-1}$ . Besides, increasing  $\omega$  from  $26.2 \text{ s}^{-1}$  to  $104.6 \text{ s}^{-1}$  at  $r_o = 0.1 \text{ m}$  results in the increase of  $V_{AC}$  from  $10.33 \text{ m}\cdot\text{s}^{-1}$  to  $41.32 \text{ m}\cdot\text{s}^{-1}$ .

The choice of the sector for the supply of fertilisers onto the spreading disk affects but little the value of  $V_{AC}$ . Thus, at  $R = 0.34 \text{ m}$ ,  $f_f = 0.55$ ,  $\alpha = 90^\circ$ ,  $r_o = 0.2 \text{ m}$ ,  $\gamma_o = 1^\circ$ ,  $\omega = 104.6 \text{ s}^{-1}$  the values of  $V_{AC}$  will be the following: sector I –  $V_{AC} = 40.74 \text{ m}\cdot\text{s}^{-1}$ ; sector II –  $V_{AC} = 40.78 \text{ m}\cdot\text{s}^{-1}$ ; sector III –  $V_{AC} = 40.75 \text{ m}\cdot\text{s}^{-1}$ ; sector IV –  $V_{AC} = 40.74 \text{ m}\cdot\text{s}^{-1}$ .



**Fig. 3.** – Dependence of absolute velocity  $V_{ac}$  of a fertiliser particle leaving the surface of the spreading disk on the radius  $r_o$  of its supply (fertilisers are supplied onto the surface of the disk in sector II at  $R = 0.34$  m,  $f_f = 0.55$ ,  $\alpha = 30^\circ$ ,  $\gamma_o = 1^\circ$ ): 1 –  $\omega = 104.6$  s<sup>-1</sup>; 2 –  $\omega = 78.5$  s<sup>-1</sup>; 3 –  $\omega = 52.3$  s<sup>-1</sup>; 4 –  $\omega = 26.2$  s<sup>-1</sup>.

In order to determine the value of the angle at which fertilisers leave the spreading disk, at first it is necessary to find out the place of their leaving. Considering that the position of the blade at the moment of its contact with the fertiliser particle is known, it is expedient to use angle  $\beta_a$  of its acceleration in order to determine the place from which fertilisers leave the spreading disk. The angle of acceleration is an angle between the positions of the blade at the moment of its contact with the fertiliser particle and the same blade at the moment when fertilisers leave the disk. Let us write an equation to determine angle  $\beta_a$ :

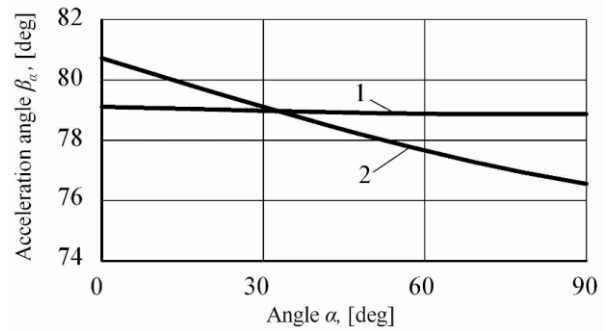
$$\beta_a = \omega t \quad (37)$$

Using expressions (35), (36) and equation (37) the impact of parameters  $\omega$ ,  $r_o$  and  $\alpha$  upon  $\beta_a$  was studied. The obtained results are presented in Fig. 4 and 5.

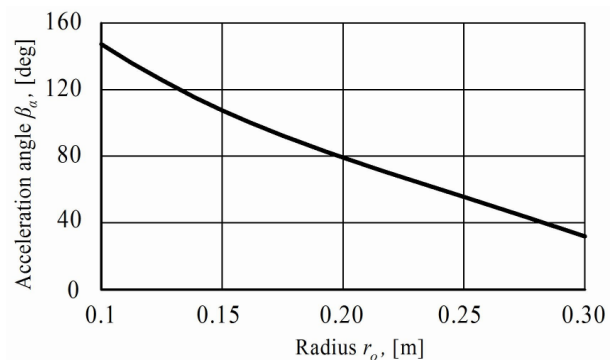
On the basis of the graphs in Fig. 4 one can draw a conclusion that at high values of  $\omega$  the impact of the inclination angle  $\alpha$  of the axis of rotation of the spreading disk upon the acceleration angle  $\beta_a$  of the fertiliser particle is insignificant. Thus, at  $\omega = 104.6$  s<sup>-1</sup> increasing  $\alpha$  from 0° to 90° results in decreased values of  $\beta_a$  from 79.13° to 78.86°.

It has been established that increasing  $r_o$  from 0.1 m to 0.3 m leads to decreased values of  $\beta_a$  from 147.53° to 32.24°. Besides, the choice of the sector for the fertiliser supply onto the surface of the disk affects a little the value of angle  $\beta_a$ . Thus, at  $R = 0.34$  m,  $f_f$

$= 0.55$ ,  $\alpha = 90^\circ$ ,  $r_o = 0.2$  m,  $\gamma_o = 1^\circ$ ,  $\omega = 104.6$  s<sup>-1</sup> the values of  $\beta_a$  will be: sector I –  $\beta_a = 79.16^\circ$ ; sector II –  $\beta_a = 78.86^\circ$ ; sector III –  $\beta_a = 79.09^\circ$ ; sector IV –  $\beta_a = 79.23^\circ$ .



**Fig. 4.** – Dependence of the acceleration angle  $\beta_a$  of a fertiliser particle on angle  $\alpha$  (fertilisers are supplied onto the surface of the disk in sector II, at  $R = 0.34$  m,  $f_f = 0.55$ ,  $r_o = 0.2$  m,  $\gamma_o = 1^\circ$ ): 1 –  $\omega = 104.6$  s<sup>-1</sup>; 2 –  $\omega = 26.2$  s<sup>-1</sup>.



**Fig. 5.** – Dependence of the acceleration angle  $\beta_a$  of a fertiliser particle on radius  $r_o$  of its supply onto the disk (fertilisers are supplied onto the surface of the disk in sector II,  $R = 0.34$  m,  $f_f = 0.55$ ,  $\omega = 104.6$  s<sup>-1</sup>,  $\alpha = 30^\circ$ ,  $\gamma_o = 1^\circ$ ).

The use of the revealed dependencies and the methodology for the determination of the angle at which the fertiliser particle leaves the surface of the spreading disk provides a possibility to obtain initial data for the estimation of the distribution distance of a fertiliser particle by the spreading disk. In its turn, determination of the distribution distance of the fertiliser particle from the centrifugal spreading tool makes it possible to substantiate other rational parameters and modes of operation of the centrifugal spreading tool.



## CONCLUSIONS

1. New theoretical dependencies have been revealed which describe the movement of a particle of mineral fertilisers along the radially situated blade of the centrifugal spreading tool the axis of rotation of which is arranged at an angle to the horizontal plane.
2. The results of numerical estimations on the basis of the newly obtained formulae allow to evaluate the

degree of impact of individual parameters of the inclined centrifugal spreading tool for the distribution of fertilisers by the value of absolute velocity of the particle leaving the disk and to obtain initial data for the estimation of the distribution distance of mineral fertilisers.

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## Corresponding author:

Semjons Ivanovs, Latvia University of Agriculture, Latvia, e-mail: semjons@apollo.lv