GENETIC OPERATORS EFFECT ON STACKING SEQUENCE OPTIMIZATION

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Abstract
In the study, stacking sequence optimization of a composite beam is carried out for various boundary conditions. Genetic algorithm (GA) is used as an optimization technique and genetic operators such as crossover and mutation ratio are changed throughout the optimization process. In the minimization of deflection parameters due to the corresponding stacking sequences, the effect of genetic operators are investigated for various boundary conditions, number of layers and fiber angle increments. A unified shear deformation theory is used in analytical solution. Minimum deflection parameters and corresponding stacking sequences are compared with the analytical solution. The variation of deflection parameters with respect to the number of generation are presented for different crossover and mutation ratios.

Key words: composite beam, deflection, genetic algorithm, optimization.

INTRODUCTION

Composite materials have gained popularity in modern structures that need to be light-weight and yet strong enough to carry various loading types for different boundary conditions. Because of high elasticity modulus and strength, composite materials are popular than any conventional materials, besides, by changing the design parameters such as stacking sequence, geometrical properties, matrix and fiber materials, structures can be designed variously in order to satisfy the desired mechanical properties. Too many design parameters in optimization problem will bring about too many solutions with it. Instead of dealing with the whole possible solutions and obtain the optimum, using an evolutionary optimization technique would be inevitable. Genetic algorithm is widely used in the design of laminated composite structures with too many variables and different fitness functions. LOPEZ ET AL. (2009) developed a GA to pursue the optimization of hybrid laminated composite structures. They considered the fiber orientation, material and total number of plies as design variables and investigated the maximum stresses. SCIUVA ET AL. (2003), optimized the stacking sequences by genetic algorithm in order to obtain the minimum buckling load and minimum weight. LEE ET AL. (2012), used a parallel evolutionary algorithm in multi-objective optimization of the stacking sequences in composite plates by choosing fiber type, thickness and orientation angles as design parameters for each layer. ALMEIDA AND AWRUCH (2009), used GA in the optimization of composite structures by using genetic operators effectively in the design process and compared the results with the ones obtained by a finite element method. BRIGHENTI (2005), investigated the optimum fiber orientation of fiber reinforced composite materials in order to optimize a certain fitness function by use of GA.

In this study, on the basis of a unified shear deformation beam theory, the minimum deflection parameters and corresponding stacking sequences are obtained for various boundary conditions and compared with the ones obtained by GA. The variation of deflection parameters with respect to the number of generations are illustrated for different crossover and mutation ratios.

MATERIALS AND METHODS

During the optimization process, a beam with rectangular cross-section, unit width, length of “L”, thickness of “h” is considered (Fig. 1). The coordinate system is placed in the mid-plane where $0 \le x \le L$ and $-h/2 \le z \le h/2$. The beam is assumed to be constructed of linearly elastic layers and under of a uniform distributed transverse load “$q(x)$” on its top plane.

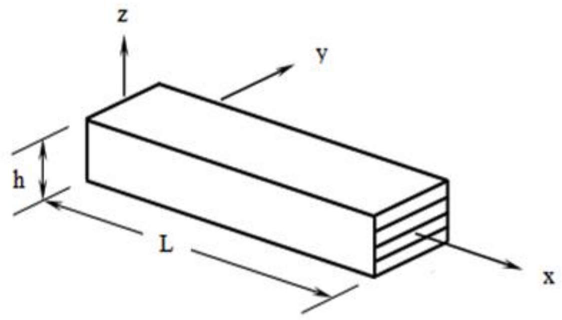

Fig. 1. – Beam geometry and coordinate axis

A uniform shear deformation beam theory is used at which classical and shear deformation theories can be obtained as a special case by use of a shape function. The displacement fields used are in parallel with general shear deformation theory (SOLDATOS AND TIMARCI, 1993) and given as follows:

$$U(x,z) = u(x) - z\, w_{,x} + \emptyset(z)\, u_1(x) \qquad W(x,z) = w(x) \tag{1}$$

The terms of "*U*" and "*W*" represent the displacement fields with respect to *x* and *z* axis where "*u*", "u_1" and "*w*" represent the displacement fields of the mid-plane. ",*x*" denotes the differentiation with respect to *x*. "Ø(*z*)" is the shape function and chosen as a cubic function of layer thickness (KARAÇAM, 2005). The displacement fields to the following kinematic relations:

$$\varepsilon_x = u_{,x} - z w_{,xx} + z\left(1 - \frac{4z^2}{3h^2}\right) u_{1,x} \qquad \gamma_{xz} = \left(1 - \frac{4z^2}{h^2}\right) u_1 \tag{2}$$

Hooke's Law for the stresses of each k[th] layer is given as follows:

$$\begin{bmatrix} \sigma_x^{(k)} \\ \tau_{xz}^{(k)} \end{bmatrix} = \begin{bmatrix} Q_{11}^{(k)} & 0 \\ 0 & Q_{55}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \gamma_{xz} \end{bmatrix} \tag{3}$$

The terms of "$Q_{ij}^{(k)}$" are the transformed reduced stiffnesses depending on material properties such as elasticity modulus (E_{ij}), shear modulus (G_{ij}) and Poisson ratio (υ_{ij}) (JONES, 1975). Using stress-strain relations into force and moment definitions (TIMARCI AND SOLDATOS, 1995), the constitutive equations are obtained as follows:

$$\begin{bmatrix} N_x \\ M_x \\ M_x^a \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & B_{111} \\ B_{11} & D_{11} & D_{111} \\ B_{111} & D_{111} & D_{1111} \end{bmatrix} \begin{bmatrix} u_{,x} \\ -w_{,xx} \\ u_{1,x} \end{bmatrix}, \qquad Q_x^a = A_{55}\, u_1 \tag{4}$$

The superscript "*a*" stands for the shear deformation effects. "A_{ij}", "B_{ij}" and "D_{ij}" are the material rigidities that correspond to extensional, coupling and bending rigidities respectively. Rigidities with more than two indices correspond to shear deformation beam theories where the ones with two indices correspond to classical beam theories. "Q_x^a" is the resultant force. The governing equations used in the analysis of a laminated composite beam under a uniform distributed load are given as follows:

$$N_{x,x} = 0 \qquad M_{x,xx} = q(x) \qquad M_{x,x}^a - Q_x^a = 0 \tag{5}$$

By use of constitutive equations into the governing equations, a set of differential equations and their derivatives depending on material rigidities and displacement functions are obtained as follows:

$$\begin{aligned} A_{11}u_{,xx} - B_{11}w_{,xxx} + B_{111}u_{1,xx} &= 0 \\ B_{11}u_{,xxx} - D_{11}w_{,xxxx} + D_{111}u_{1,xxx} &= q(x) \\ B_{111}u_{,xx} - D_{111}w_{,xxx} + D_{1111}u_{1,xx} - A_{55}u_1 &= 0 \end{aligned} \tag{6}$$

Boundary conditions at both edges of the beam when *x*=0 and *x*=*L* are given respectively for simply supported, cantilever and free cases (SOLDATOS AND TIMARCI, 1993):

$$\begin{aligned} N_x = w = M_x = M_x^a &= 0 \\ u = w = w_{,x} = u_1 &= 0 \\ N_x = M_x = M_{x,x} = M_x^a &= 0 \end{aligned} \tag{7}$$

By integrating and solving equations simultaneously and using the boundary conditions at both edges, three unknown displacement functions are obtained with eight integration constants (C_k). The displacement functions are given as below:

$$\begin{aligned} u_1(x) &= C_1 e^{-px} + C_2 e^{px} - (qx + C_3)\frac{D_{111}}{A_{55}D_{11}} = 0 \\ u(x) &= -\frac{B_{111}}{A_{11}} u_1(x) + C_7 x + C_8 = 0 \\ w(x) &= \frac{D_{111}}{pD_{11}}\left[-C_1 e^{-px} + C_2 e^{px} - \frac{p}{A_{55}}\left(\frac{qx^2}{2} + C_3 x\right)\right] + \frac{1}{D_{11}}\left(\frac{qx^4}{24} + C_3\frac{x^3}{6}\right) + C_4\frac{x^2}{2} + C_5 x + C_6 = 0 \end{aligned} \tag{8}$$

$$p = \sqrt{\frac{-A_{55}A_{11}D_{11}}{D_{111}^2 A_{11} - D_{1111}A_{11}D_{11}}}$$

In the study, genetic algorithm is used as an optimization technique. Genetic algorithm is an evolutionary optimization technique using Darwin's principal of "survival of the fittest". It's a guided random search technique that works on a population of designs. The principals of GA are firstly proposed by HOLLAND

(1995) in optimization problems. Early applications of GA to structural optimization are due to GOLDBERG (1989) and HAJELA (1990). GA starts with a random initial population of possible design alternatives. If the population is suitably large enough, GA is not at the risk of being stuck in a local optimum (GHIASI ET AL., 2009). They are also derivative-free optimization methods as they don't need functional derivative information. By using such algorithms in the optimization, robustness and flexibility can easily be obtained (GOLDBERG, 1989). A major limitation of traditional GA's, especially in larger populations, is the time waste to distinguish the convenient fitness function. In order to prevent this, genetic operators such as reproduction, crossover and mutation are applied to the algorithm. In the optimization process, stacking sequences are used as the design parameters. Initially, fiber orientation angles of each layer are coded in order to build up the stacking sequences of composite beam which also correspond to the chromosomes of each generation in GA. After the coding operation, a random population of stacking sequences which is also called as the initial population is generated. Reproduction is the first genetic operator used in genetic algorithm at which the population is generated and ranked depending on the design parameters. Besides, by use of elitism in reproduction, the quality of the population can be increased by copying the best individuals of each generation into the next generation. Then crossover is applied to generate new individuals by swapping one or more genes which correspond to the fiber orientation angle of a chromosome. Mutation is another genetic operator used in GA. It's generally used to maintain the genetic diversity from one generation to another. It basically depends on altering one or more genes of a chromosome among the population. The mutation ratio is generally chosen between 0.01 and 0.001 in order to prevent the negative influence (GOLDBERG, 1989). The genetic operators are applied until the stopping criteria or the targeted value is obtained. A simple genetic algorithm flowchart is given in Fig. 2.

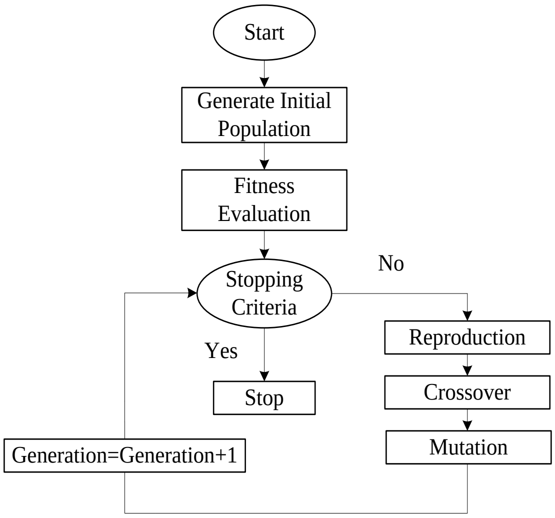

Fig. 2. – A simple genetic algorithm flowchart

RESULTS AND DISCUSSION

In the study, in order to ensure the reliability of the theory used, the deflection parameters are compared with KHDEIR AND REDDY (1997) for classical and parabolic shear deformation beam theories initially. In Tab. 1, non-dimensional deflection parameters are presented for classical and parabolic shear deformation beam theories and various boundary conditions. For parabolic shear deformation beam theory, the results are very close to reference values for clamped-clamped (C-C) and simply supported (S-S) boundary conditions and the same for clamped-free (C-F) boundary condition. As for the classical beam theory, all of the non-dimensional deflection parameters are the same with reference values.

Tab. 1. – Non-dimensional deflection parameters for various boundary conditions

L/h	Theory	C-C		C-F		S-S	
		Model	Ref.	Model	Ref.	Model	Ref.
10	PSDT	0.5316	0.5320	3.4550	3.4550	1.0663	1.0960
	CBT	0.1290	0.1290	2.1980	2.1980	0.6460	0.6460
50	PSDT	0.1469	0.1470	2.2510	2.2510	0.6645	0.6650
	CBT	0.1290	0.1290	2.1980	2.1980	0.6460	0.6460

In the study presented, the composite beam is assumed to be constructed of graphite/epoxy and under of a uniform distributed load where $q(x)$=1000 N/m. Mechanical properties of graphite/epoxy are given as follows (KARAMA ET AL., 2003):

$$E_{11}=241.5 \text{ GPa}, E_{22}=E_{33}=18.89 \text{ GPa}$$
$$G_{23}=3.45 \text{ GPa}, G_{12}=G_{13}=5.18 \text{ GPa} \quad (9)$$
$$\upsilon_{23}=0.25, \upsilon_{12}=\upsilon_{13}=0.24$$

The population is considered to be consisted of 100 individuals and fiber orientation angles (θ) are assumed to be changing with an increment of 10° and 30° where 0° ≤ θ ≤ 90°. The minimization of the deflection parameter is carried out at certain points of the beam for various boundary conditions where the maximum deflections may occur. For C-C and S-S boundary conditions, the deflection parameters are calculated in the middle of beam where $x=L/2$ and for (C-F) boundary condition at the free edge of the beam where $x=L$.

Tab. 2. – The minimum deflection parameters and corresponding stacking sequences for various boundary conditions and number of layers

Boundary Conditions	Number of Layers	Exact Solution		GA Solution	
		w (×10^{-5}) [m]	Stacking Sequence	w (×10^{-5}) [m]	Stacking Sequence
C-C	3	1.7523	0°/0°/0°	1.7523	0°/0°/0°
C-F		49.5405	0°/90°/90°	49.5405	0°/90°/90°
S-S		10.2264	0°/0°/30°	10.2264	0°/0°/30°
C-C	4	1.7523	0°/0°/0°/0°	1.7523	0°/0°/0°/0°
C-F		47.8485	0°/60°/60°/30°	47.8485	0°/60°/60°/30°
S-S		8.4360	0°/0°/30°/0°	8.4360	0°/0°/30°/0°
C-C	5	1.6772	0°/30°/90°/30°/30°	1.6772	0°/30°/90°/30°/30°
C-F		35.7074	0°/0°/90°/90°/30°	35.7074	0°/0°/90°/90°/30°
S-S		8.5742	0°/0°/0°/30°/0°	8.5742	0°/0°/0°/30°/0°

Tab. 3. – The minimum deflection parameters and corresponding stacking sequences for various boundary conditions and number of layers

Boundary Conditions	Number of Layers	Exact Solution		GA Solution	
		w (×10^{-5}) [m]	Stacking Sequence	w (×10^{-5}) [m]	Stacking Sequence
C-C	3	1.7523	0°/0°/0°	1.7523	0°/0°/0°
C-F		42.8886	0°/70°/40°	42.8886	0°/70°/40°
S-S		8.4238	0°/0°/10°	8.4238	0°/0°/10°
C-C	4	0.9943	0°/10°/70°/30°	0.9943	0°/10°/70°/30°
C-F		35.9179	0°/0°/70°/20°	35.9179	0°/0°/70°/20°
S-S		8.2277	0°/0°/10°/0°	8.2277	0°/0°/10°/0°
C-C	5	0.0110	40°/0°/30°/60°/90°	0.0365	0°/70°/40°/50°/70°
C-F		31.9941	0°/0°/70°/90°/20°	31.9941	0°/0°/70°/90°/20°
S-S		8.2456	0°/0°/0°/10°/0°	8.2456	0°/0°/0°/10°/0°

After a random initial population being generated, the deflection parameters with corresponding stacking sequences are ranked from minimum to maximum along 50 generation. After 50th generation, the minimum deflection parameters and corresponding stacking sequences are obtained for various boundary conditions. In Tab. 2 and Tab. 3, minimum deflection parameters (w) and corresponding stacking sequences are given for various boundary conditions and number of layers respectively. In Tab. 2, the fiber orientation angles are changed with an increment of 30° and the deflection parameters obtained by genetic algorithm are compared with the exact solution. For all boundary conditions and number of layers, the identical deflection parameters are obtained for both exact and GA solution.

In Tab. 3, the fiber orientation angles are changed with an increment of 10°. Although the results are identical for 3 and 4 number of layers for both solution, there is a minor difference in deflection parameters for 5 number of layers due to the excessive number of possible stacking sequences. The convergence to minimum can be increased by an efficient use of genetic operators or a convenient number of population and generation.

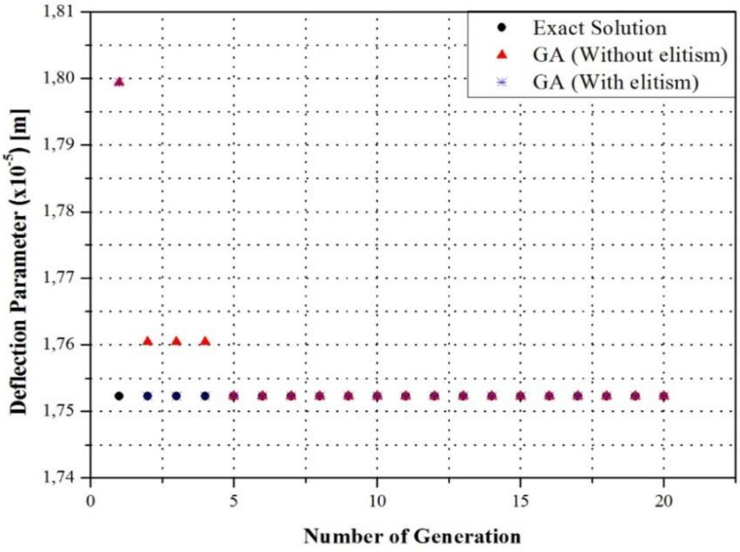

Fig. 3. – The variation of deflection parameters with respect to the generation for C-C

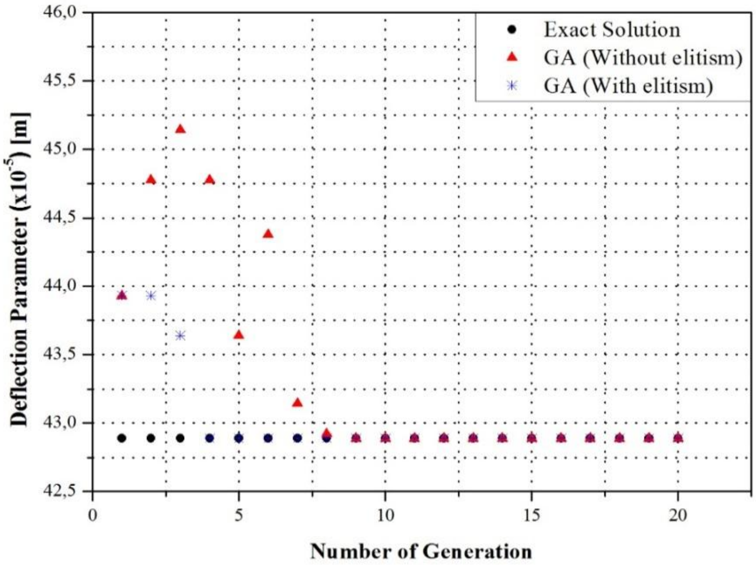

Fig. 4. – The variation of deflection parameters with respect to the generation for C-F

During the optimization process, initially the exact solution which gives the minimum deflection is performed. Stacking sequence optimization is carried out for two cases. In the first case, deflection parameters are obtained without using elitism, then elitism is included into the algorithm. In both cases, the crossover ratio (CR) and mutation ratio (MR) are chosen as 0.2 and 0.1 respectively. In Fig. 3, Fig. 4 and Fig. 5, the variation of deflection parameters with respect to the number of generation are illustrated for various boundary conditions. The beam is assumed to be constructed of three layers with a fiber orientation angle increment of 10°. Although the minimum values are obtained for both cases, they are obtained in earlier generations by use of elitism in 2nd, 4th and 3rd generation for C-C, C-F and S-S boundary condition respectively.

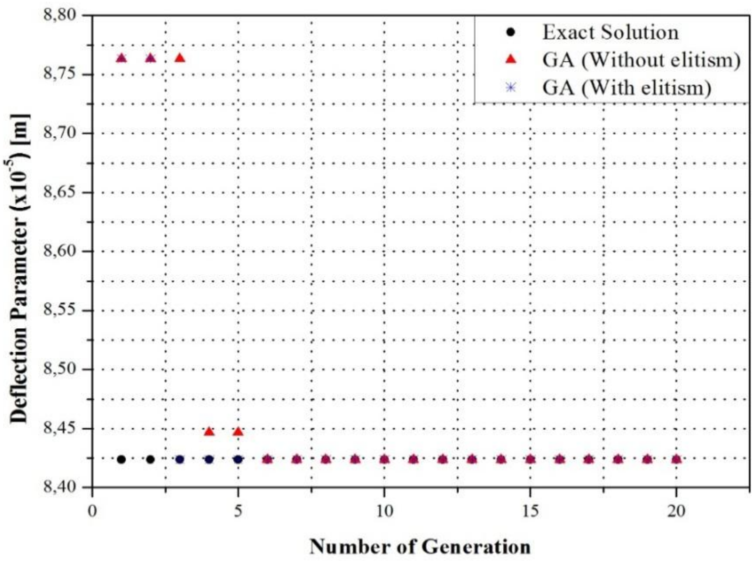

Fig. 5. – The variation of deflection parameters with respect to the generation for S-S

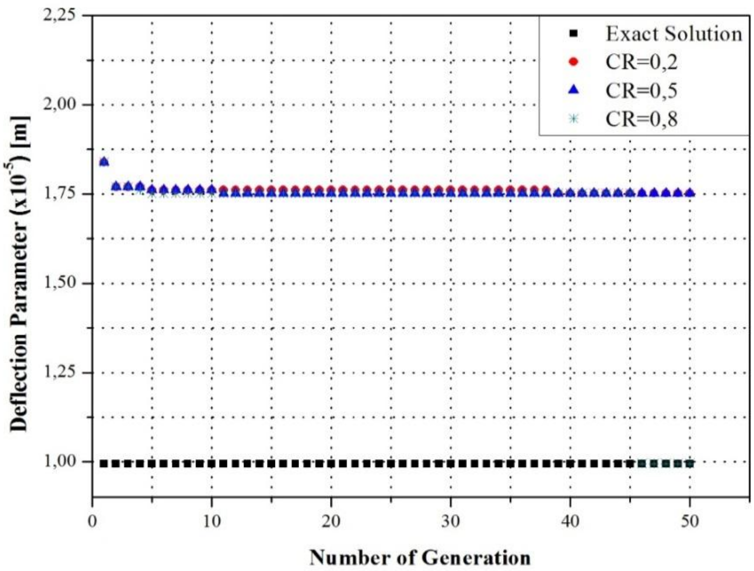

Fig. 6. – The variation of deflection parameters with respect to the generation for C-C

In Fig. 6, Fig. 7 and Fig. 8, the variation of deflection parameters with respect to the number of generation are illustrated for various crossover ratios and boundary conditions. The beam is assumed to be constructed of four layers with a fiber orientation angle increment of 10°. Firstly, exact solution of the problem is performed for each boundary condition. In the optimization stage, mutation ratio is chosen as 0.01 and different crossover ratios are considered such as 0.2, 0.5 and 0.8. These crossover ratios correspond to various number of swapping layers. For instance, a beam with four layers corresponds to an individual of four genes. Only one and the first gene of an individual is changed with the consecutive one for a crossover ratio of 0.2. The first two and three genes of the consecutive individuals are changed respectively for crossover ratio of 0.5 and 0.8. Due to a number of possible stacking sequences, elitism is included into the algorithm in order to increase the convergence and prevent local minimum. Therefore, the minimum deflection parameters are generally obtained in recent generations. Although the minimum value is obtained at CR=0.8 in C-C boundary condition, they are obtained at CR=0.5 for C-F and S-S boundary conditions respectively. The stacking sequences with a crossover ratio of 0.5 have better results and they converge faster than the others for given conditions.

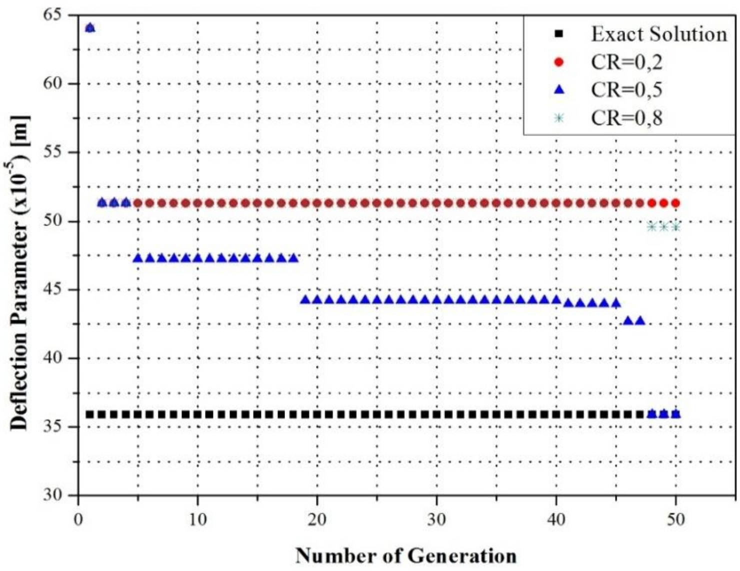

Fig. 7. – The variation of deflection parameters with respect to the generation for C-F

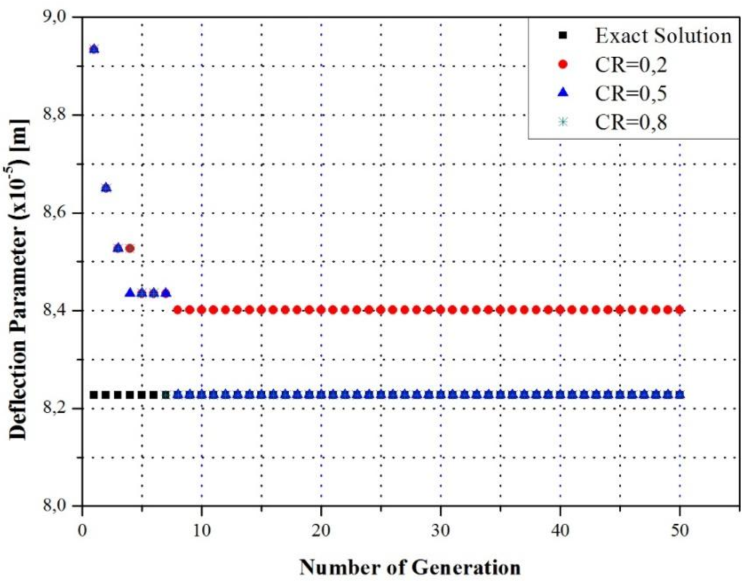

Fig. 8. – The variation of deflection parameters with respect to the generation for S-S

The reliability of the algorithm can be expressed as a percentage of minimum deflection parameters equal to the optimum values. The values in Tab. 2 and 3 are obtained among a large number of possible solutions depending on number of layers and fiber angle increment. Although there are 64, 256 and 1024 possible stacking sequences for an increment of 30° for 3, 4 and 5 layers respectively, there are 1000, 10000 and 100000 values for an increment of 10°. Since all the deflection parameters are the same with exact solutions in Tab. 2, % 100 reliability is satisfied in all boundary conditions. 89% reliability is obtained in Tab. 3 due to the excessive number of possible stacking sequences.

CONCLUSIONS

In the study, in order to minimize the deflection parameters, stacking sequence optimization of laminated composite beam is carried out for various boundary conditions and genetic operators by use of genetic algorithm. The fiber angles are assumed to be changing between 0° and 90° with an increment of 10°, 30°. Since it is very difficult to produce and analyze a composite beam consisting of specific stacking sequences, material and geometrical properties in real life, the algorithm developed in this study will be helpful in the development and analysis of a prototype without any cost. On the other hand, too many design parameters which correspond to a number of possible solutions will increase the processing time in the optimization of composite structures. Owing to evolving individuals in each generation, the algorithm developed will decrease the processing time. Since the composite beams are quite thin, the same results are obtained both for exact and GA solution. Depending on the applicability of technique to various type of optimization problems and regardless of the number of design parameters, it is concluded that using an evolutionary optimization technique in the design of composite beams will be inevitable.

REFERENCES

1. ALMEIDA, F. S., AWRUCH, A. M.: Design optimization of composite laminated structures using genetic algorithms and finite element analysis, Composite Structures, 88, 2009: 443-454.
2. BRIGHENTI, R.: Fibre distribution optimisation in fibre-reinforced composites by a genetic algorithm, Composite Structures, 71, 2005: 1-15.
3. GHIASI, H., PASINI, D., LESSARD, L.: Optimum stacking sequence design of composite materials, Part I: Constant stiffness design, Composite Structures, 90, 2009: 1-11.
4. GOLDBERG, D. E.: Genetic algorithm in search, optimization and machine learning Addison-Wesley Publishing, Inc., Reading, Massachusetts, USA, 1989.
5. HAJELA, P.: Genetic search an approach to the nonconvex optimization problem, AIAA Journal, 26 (7), 1990: 1205-1210.
6. HOLLAND, J. H.: Adaptation in natural and artificial systems: An introductory analysis with applications to biology, control and artificial intelligence, MIT Press, Cambridge, MA, USA, 1995.
7. JONES, R. M.: Mechanics of composite materials, McGraw-Hill, New York, USA, 1975.
8. KARAÇAM, F.: Bending Analysis of Laminated Composite Beams (in Turkish) [M.Sc. Thesis], Institute of Science, Department of Mechanical Engineering, Trakya University, Edirne, TURKEY, 2005, 47 p.
9. KARAMA, M., AFAQ, K. S., MISTOU, S.: Mechanical behaviour of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity, International Journal of Solids and Structures, 40, 2003: 1525-1546.
10. KHDEIR, A. A., REDDY, J. N.: An exact solution for the bending of thin and thick cross-ply laminated beams, Composite Structures, 37, 1997: 195-203.
11. LEE, D. S., MORILLO, C., BUGEDA, G., OLLER, S., ONATE, E.: Multilayered composite structure design optimisation using distributed/parallel multi-objective evolutionary algorithms, Composite Structures, 94, 2012: 1087-1096.
12. LOPEZ, R. H., LUERSEN, M. A., CURSI, J. E. S.: Optimization of hybrid laminated composites using a genetic algorithm, Journal of the Brazilian Society of Mechanical Sciences and Engineering, 31 (3), 2009: 269-278.
13. SCIUVA, D. M., GHERLONE, M., LOMARIO, D.: Multiconstrained optimization of laminated and sandwich plates using evolutionary algorithms and higher-order plate theories, Composite Structures, 50, 2003: 149-154.
14. SOLDATOS, K. P., TIMARCI, T.: A unified formulation of laminated composite, shear deformable, five-degrees-of-freedom cylindrical shell theories, Composite Structures, 25, 1993: 165-171.
15. TIMARCI, T., SOLDATOS, K. P.: Comparative dynamic studies for symmetric cross-ply circular cylindrical shells on the basis of a unified shear deformable shell theory, Journal of Sound and Vibration, 187 (4), 1995: 609-624.

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